

# Intrinsic problems of gravitational baryogenesis

E.V. Arbuzova<sup>1,2,\*</sup> and A.D. Dolgov<sup>1,3,†</sup>

<sup>1</sup>*Novosibirsk State University, Novosibirsk, 630090, Russia*

<sup>2</sup>*Department of Higher Mathematics, Dubna State University, 141980 Dubna, Russia*

<sup>3</sup>*ITEP, Bol. Cheremushkinsaya ul., 25, 117259 Moscow, Russia*

The modified gravitational equations of motion, induced by the curvature dependent term in the gravitational baryogenesis scenario, are derived. It is shown that this term leads to the fourth order equations for curvature instead of the usual algebraic ones of General Relativity. The fourth order gravitational equations are generically unstable with respect to small perturbations. The solutions may be stabilized by the effects of the non-linear in curvature terms. However, the magnitude of the stabilized curvature scalar would be much larger than the usual cosmological curvature.

## I. INTRODUCTION

Different scenarios of baryogenesis are based, as a rule, on three well known Sakharov principles [1]:

1. Non-conservation of baryonic number.
2. Breaking of symmetry between particles and antiparticles.
3. Deviation from thermal equilibrium. For the details see e.g. the reviews [2, 3].

However, as it is mentioned in the review [2], none of these conditions is obligatory. Of particular interest is the scenario of spontaneous baryogenesis (SBG) [4], which can proceed in thermal equilibrium, moreover, it is usually most efficient in thermal equilibrium. The term "spontaneous" is related to spontaneous breaking of underlying symmetry of the theory. It is assumed that in the unbroken phase the Lagrangian is invariant with respect to the global  $U(1)$ -symmetry, which ensures conservation of the total baryonic number. This symmetry is supposed to be spontaneously broken and in the broken phase the Lagrangian density acquires the term

$$\mathcal{L}_{SB} = (\partial_\mu \theta) J_B^\mu, \quad (1.1)$$

where  $\theta$  is Goldstone field and  $J_B^\mu$  is the baryonic current of matter fields.

For the spatially homogeneous field  $\theta = \theta(t)$  the Lagrangian (1.1) is reduced to  $\mathcal{L}_{SB} = \dot{\theta} n_B$ , where  $n_B \equiv J_B^0$  is the baryonic number density of matter, so it is tempting to identify  $(-\dot{\theta})$  with the baryonic chemical potential,  $\mu$ , of the corresponding system. The identification of  $\dot{\theta}$  with chemical potential is questionable and depends upon the representation chosen for the fermionic fields, as it is argued in refs. [5, 6], but still the scenario is operative and presents a beautiful possibility to create an excess of particles over antiparticles in the universe.

Later the idea of gravitational baryogenesis (GBG) was put forward [7], where the scenario of SBG was modified by an introduction of the coupling of baryonic current to the derivative of the curvature scalar  $R$ :

$$\mathcal{L}_{GBG} = \frac{1}{M^2} (\partial_\mu R) J_B^\mu, \quad (1.2)$$

where  $M$  is a constant parameter with dimension of mass. More references on GBG can be found in [8].

In the presented work we demonstrate that GBG in classical general relativity (GR) suffers from strong gravitational instability, because the usual second order GR equations turn into higher order equations due to contribution from  $\mathcal{L}_{GBG}$  (1.2). Examples of similar gravitational instability in  $F(R)$  gravity have been demonstrated in refs. [9].

## II. EQUATIONS OF MOTION

Let us start from the model where baryonic number is carried by scalar field  $\phi$  with potential  $U(\phi, \phi^*)$ . An example with baryonic current of fermions will be considered elsewhere.

The action of the scalar model has the form:

$$A = \int d^4x \sqrt{-g} \left[ \frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2} (\partial_\mu R) J^\mu - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + U(\phi, \phi^*) \right] - A_m, \quad (2.1)$$

---

\*Electronic address: arbuzova@uni-dubna.ru

†Electronic address: dolgov@fe.infn.it

where  $m_{Pl} = 1.22 \cdot 10^{19}$  GeV is the Planck mass,  $A_m$  is matter action, and  $J^\mu = g^{\mu\nu} J_\nu$ ,  $g^{\mu\nu}$  is the metric tensor of the corresponding space-time.

### A. Current: version 1

If the potential  $U(\phi)$  is not invariant with respect to  $U(1)$ -rotation,  $\phi \rightarrow \exp(i\beta)\phi$ , the baryonic current defined in the usual way

$$J_{1\mu} = iq(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) \quad (2.2)$$

is not conserved. Here  $q$  is the baryonic number of  $\phi$  and for brevity we omitted index  $B$  in current  $J_\mu$ .

The corresponding equations of motion for gravitational field have the form:

$$\begin{aligned} \frac{m_{Pl}^2}{16\pi} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) - \frac{1}{M^2} \left( [R_{\mu\nu} - (D_\mu D_\nu - g_{\mu\nu} D^2)] D_\alpha J_1^\alpha + \frac{1}{2} g_{\mu\nu} J_1^\alpha D_\alpha R - \frac{1}{2} (J_{1\nu} D_\mu R + J_{1\mu} D_\nu R) \right) \\ - \frac{1}{2} (D_\mu \phi D_\nu \phi^* + D_\nu \phi D_\mu \phi^*) + \frac{1}{2} g_{\mu\nu} (D_\alpha \phi D^\alpha \phi^* - U(\phi)) = \frac{1}{2} T_{\mu\nu}, \end{aligned} \quad (2.3)$$

where  $D_\mu$  is the covariant derivative in metric  $g_{\mu\nu}$  (of course, for scalars  $D_\mu = \partial_\mu$ ) and  $T_{\mu\nu}$  is the energy-momentum tensor of matter obtained from action  $A_m$ .

Taking trace of equation (2.3) with respect to  $\mu$  and  $\nu$  we obtain:

$$\frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2} [(R + 3D^2) D_\alpha J_1^\alpha + J_1^\alpha D_\alpha R] - D_\alpha \phi D^\alpha \phi^* + 2U(\phi) = -\frac{1}{2} T^\mu_\mu. \quad (2.4)$$

The equation of motion for the field  $\phi$  is:

$$D^2 \phi + \frac{\partial U}{\partial \phi^*} = -\frac{iq}{M^2} (2D_\mu R D^\mu \phi + \phi D^2 R). \quad (2.5)$$

According to definition (2.2) the current divergence is:

$$D_\mu J_1^\mu = \frac{2q^2}{M^2} [D_\mu R (\phi^* D^\mu \phi + \phi D^\mu \phi^*) + |\phi|^2 D^2 R] + iq \left( \phi \frac{\partial U}{\partial \phi} - \phi^* \frac{\partial U}{\partial \phi^*} \right). \quad (2.6)$$

If  $U = U(|\phi|)$ , the last term in this expression disappears.

### B. Current: version 2

If  $U = U(|\phi|)$  and so the theory is invariant with respect to the phase rotation,  $\phi \rightarrow \exp(i\beta)\phi$  with constant  $\beta$ , then according to the Noether theorem there must exist the conserved current, which has the form

$$J_{2\mu} = iq(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) - \frac{2q^2}{M^2} |\phi|^2 D_\mu R. \quad (2.7)$$

The equation of motion for the field  $\phi$  is modified as:

$$D^2 \phi + \frac{\partial U}{\partial \phi^*} = -\frac{iq}{M^2} (2D_\mu R D^\mu \phi + \phi D^2 R) + \frac{2q^2}{M^4} \phi D_\mu R D^\mu R, \quad (2.8)$$

and the current divergence is now

$$D_\mu J_2^\mu = iq \left( \phi \frac{\partial U}{\partial \phi} - \phi^* \frac{\partial U}{\partial \phi^*} \right). \quad (2.9)$$

It is easy to see, that the r.h.s. of this equation vanishes, if  $U = U(|\phi|)$ .

The corresponding equations of motion for gravitational field acquire additional terms and have the form:

$$\begin{aligned} \frac{m_{Pl}^2}{16\pi} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) - \frac{1}{M^2} \left( [R_{\mu\nu} - (D_\mu D_\nu - g_{\mu\nu} D^2)] D_\alpha J_2^\alpha + \frac{1}{2} g_{\mu\nu} J_2^\alpha D_\alpha R - \frac{1}{2} (J_{2\nu} D_\mu R + J_{2\mu} D_\nu R) \right) \\ - \frac{2q^2}{M^4} [R_{\mu\nu} - (D_\mu D_\nu - g_{\mu\nu} D^2)] D_\alpha (|\phi|^2 D^\alpha R) \\ - \frac{1}{2} (D_\mu \phi D_\nu \phi^* + D_\nu \phi D_\mu \phi^*) + \frac{1}{2} g_{\mu\nu} (D_\alpha \phi D^\alpha \phi^* - U(\phi)) = \frac{1}{2} T_{\mu\nu}. \end{aligned} \quad (2.10)$$

We have verified the transversality of the l.h.s. of Eqs. (2.3) and (2.10) by taking the covariant derivative  $D_\mu$ .

Taking the trace of equation (2.10) with respect to  $\mu$  and  $\nu$  we obtain:

$$\begin{aligned} \frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2} [(R + 3D^2)D_\alpha J_2^\alpha + J_2^\alpha D_\alpha R] + \frac{2q^2}{M^4}(R + 3D^2)D_\alpha(|\phi|^2 D^\alpha R) \\ - D_\alpha \phi D^\alpha \phi^* + 2U(\phi) = -\frac{1}{2}T_\mu^\mu. \end{aligned} \quad (2.11)$$

### III. SOLUTION IN FRW BACKGROUND

Let us consider solution of the presented above equations of motion in cosmology. The spatially flat cosmological FRW background metric has the form:

$$ds^2 = dt^2 - a^2(t)d\mathbf{r}^2. \quad (3.1)$$

In homogeneous case equation for the curvature scalar (2.4) takes the form:

$$\frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2} [(R + 3\partial_t^2 + 9H\partial_t)D_\alpha J_1^\alpha + \dot{R} J_1^0] = -\frac{1}{2}T^{(tot)}, \quad (3.2)$$

where  $H = \dot{a}/a$  is the Hubble parameter and  $T^{(tot)}$  is the trace of the energy-momentum tensor of matter including contribution from  $\phi$ -field. In thermal cosmological plasma

$$T^{(tot)} = \varrho - 3P, \quad (3.3)$$

where  $\varrho$  and  $P$  are respectively energy and pressure densities of plasma. For relativistic plasma  $\varrho = \pi^2 g_* T^4/30$  with  $T$  and  $g_*$  being respectively the plasma temperature and the number of particle species in the plasma. The Hubble parameter is expressed through  $\varrho$  as  $H^2 = 8\pi\varrho/(3m_{Pl}^2) \sim T^4/m_{Pl}^2$ .

In equation (3.2)  $J_1^0$  is the density of baryonic number of  $\phi$ -field and the covariant divergence of current is given by expression (2.6). In the considered homogeneous case it takes the form:

$$D_\alpha J_1^\alpha = \frac{2q^2}{M^2} [\dot{R}(\phi^* \dot{\phi} + \dot{\phi} \phi^*) + (\ddot{R} + 3H\dot{R})\phi^* \phi] + iq \left( \phi \frac{\partial U}{\partial \phi} - \phi^* \frac{\partial U}{\partial \phi^*} \right). \quad (3.4)$$

Substituting this expression into Eq. (3.2) we arrive to the forth order equation with respect to  $R$  with the coefficients proportional to the bilinear combinations of  $\phi$  and  $\phi^*$ . Performing the thermal averaging described in the Appendix we find

$$\langle \phi^* \phi \rangle = \frac{T^2}{12}, \quad \langle \phi^* \dot{\phi} + \dot{\phi} \phi^* \rangle = 0. \quad (3.5)$$

Assuming that the universe expansion is slow in comparison with the variation of  $R$  (as we see in what follows) we look for the solution in the form  $R \sim \exp(\lambda t)$ . We find the characteristic polynomial for eigenvalues  $\lambda$ :

$$\lambda^4 + 6H\lambda^3 + 9H^2\lambda^2 + \mu^4 = 0, \quad (3.6)$$

where

$$\mu^4 = \frac{m_{Pl}^2 M^4}{8\pi q^2 T^2}. \quad (3.7)$$

If we neglect terms proportional to the Hubble parameter,  $H$ , we come to the simple equation:

$$\lambda^4 + \mu^4 = 0, \quad (3.8)$$

which is solved as:

$$\lambda = (-1)^{1/4} \mu. \quad (3.9)$$

There are two solutions with positive real parts of  $\lambda$ . This indicates that the curvature scalar is exponentially unstable with respect to small perturbations. The characteristic rate of rising of the perturbations is much larger than the Hubble rate,  $\lambda > H$ , according to the above made assumption. In the opposite case,  $\lambda < H$ , the instability is effectively absent. In this limit we find from Eq. (3.6)

$$\frac{|\lambda^2|}{H^2} = \frac{\mu^4}{9H^4} = \frac{225}{(2\pi)^7 q^2 g_*^2} \frac{m_{Pl}^6 M^4}{T^{10}}. \quad (3.10)$$

The condition  $|\lambda^2|/H^2 < 1$  is fulfilled e.g. at Big Bang Nucleosynthesis ( $T \sim 1$  MeV) if  $M < 10^{-32}$  MeV. So small  $M$  leads to gigantic strength of interaction (1.2) and thus to unacceptable corrections to General Relativity.

Deriving Eq. (3.6) we have omitted the quadratic in curvature terms. Their presence may in principle stabilize the solution but at the values which grossly exceed the usual magnitude of the curvature in FRW cosmology.

The equations presented here are derived for the current  $J_1$  given by Eq. (2.2), but it is easy to see that essentially the same results are valid for the current  $J_2$  (2.7).

### Acknowledgments

Our work was supported by the RSF Grant N 16-12-10037.

### Appendix A: Thermal averaging of the products of operators $\phi(x)$

Field  $\phi(x)$  can be expanded in terms of quantum creation-annihilation operators as follows:

$$\phi(x) = \int \frac{d^3q}{\sqrt{2E_q(2\pi)^3}} [a(\mathbf{q})e^{-iE_q t + i\mathbf{q}\mathbf{x}} + b^\dagger(\mathbf{q})e^{iE_q t - i\mathbf{q}\mathbf{x}}], \quad (\text{A1})$$

where  $E_q = \sqrt{q^2 + m_\phi^2}$  with  $q = |\mathbf{q}|$ . In eq. (A1)  $a$  and  $a^\dagger$ ,  $b$  and  $b^\dagger$  are the annihilation and creation operators for scalar particles and antiparticles, respectively, obeying the commutation relations

$$[a(\mathbf{q}), a^\dagger(\mathbf{q}')] = 2E_q (2\pi)^3 \delta(\mathbf{q} - \mathbf{q}'), \quad (\text{A2})$$

the same for  $b(\mathbf{q})$ .

The products of creation-annihilation operators averaged over the medium have the standard form:

$$\begin{aligned} \langle a^\dagger(\mathbf{q})a(\mathbf{q}') \rangle &= f_B(E_q)\delta^{(3)}(\mathbf{q} - \mathbf{q}'), & \langle a(\mathbf{q})a^\dagger(\mathbf{q}') \rangle &= [1 + f_B(E_q)]\delta^{(3)}(\mathbf{q} - \mathbf{q}'), \\ \langle b^\dagger(\mathbf{q})b(\mathbf{q}') \rangle &= f_{\bar{B}}(E_q)\delta^{(3)}(\mathbf{q} - \mathbf{q}'), & \langle b(\mathbf{q})b^\dagger(\mathbf{q}') \rangle &= [1 + f_{\bar{B}}(E_q)]\delta^{(3)}(\mathbf{q} - \mathbf{q}'), \end{aligned} \quad (\text{A3})$$

where  $f_{B,\bar{B}}(E_q)$  is the energy dependent boson distribution function, which may be arbitrary since we assumed only that the medium is homogeneous and isotropic. We also assumed, as it is usually done, that non-diagonal matrix elements of creation-annihilation operators vanish on the average due to decoherence. For the vacuum case  $f_{B,\bar{B}}(E) = 0$  and we obtain the usual vacuum average values of  $aa^\dagger$  and  $a^\dagger a$ , which from now on will be neglected because we are interested only in the matter effects.

Averaging the product  $\phi^*(x)\phi(x)$  we obtain

$$\begin{aligned} \langle \phi^*(x)\phi(x) \rangle &= \\ &= \int \frac{d^3q d^3q'}{2(2\pi)^3 \sqrt{E_q E_{q'}}} \langle [a^\dagger(\mathbf{q})e^{iE_q t - i\mathbf{q}\mathbf{x}} + b(\mathbf{q})e^{-iE_q t + i\mathbf{q}\mathbf{x}}] [a(\mathbf{q}')e^{-iE_{q'} t + i\mathbf{q}'\mathbf{x}} + b^\dagger(\mathbf{q}')e^{iE_{q'} t - i\mathbf{q}'\mathbf{x}}] \rangle \\ &= \int \frac{d^3q}{(2\pi)^3 E_q} f(E_q, T). \end{aligned} \quad (\text{A4})$$

In thermal equilibrium the distributions of bosons and their antiparticles with zero chemical potentials have the usual Bose-Einstein form:

$$f_B(E, T) = f_{\bar{B}}(E, T) = \frac{1}{\exp(E/T) - 1}, \quad (\text{A5})$$

where in high temperature limit we can neglect the particle mass i.e. we can assume  $E_q = q$ .

The last integral in Eq. (A4) is simply taken giving:

$$\langle \phi^*(x)\phi(x) \rangle = \frac{1}{2\pi^2} \int \frac{dq q}{e^{q/T} - 1} = \frac{T^2}{2\pi^2} \int_0^\infty \frac{dz z}{e^z - 1} = \frac{T^2}{12}. \quad (\text{A6})$$

The average value of the operator  $\langle \phi^* \dot{\phi} + \dot{\phi}^* \phi \rangle$  is zero.

- [2] A.D.Dolgov, Phys. Repts **222** (1992) No. 6;  
A.D. Dolgov, Surveys in High Energy Physics, **13** (1998) 83, hep-ph/9707419.
- [3] V.A. Rubakov, M.E. Shaposhnikov, Usp. Fiz. Nauk, **166** (1996) 493, hep-ph/9603208;  
A. Riotto, M. Trodden, Ann. Rev. Nucl. Part. Sci. **49** (1999), 35, hep-ph/9901362;  
M. Dine, A. Kusenko, Rev. Mod. Phys. **76** (2004) 1.
- [4] A. Cohen, D. Kaplan, Phys. Lett. B **199**, 251 (1987);  
A. Cohen, D. Kaplan, Nucl.Phys. B **308** (1988) 913;  
A. G. Cohen, D.B. Kaplan, A.E. Nelson, Phys.Lett. B **263** (1991) 86;  
A. G. Cohen, D.B. Kaplan, A.E. Nelson, Phys.Lett. B **336** (1994) 41, hep-ph/940634.
- [5] A.D. Dolgov, K. Freese, Phys.Rev. D **51** (1995) 2693-2702, hep-ph/9410346;  
A.D. Dolgov, K. Freese, R. Rangarajan, M. Srednicki, Phys.Rev. D **56** (1997) 6155, hep-ph/9610405.
- [6] E.V. Arbuzova, A.D. Dolgov, V.A. Novikov, Phys. Rev. D **94** (2016) 123501, arXiv:1607.01247.
- [7] H. Davoudiasl, R. Kitano, G. D. Kribs, H. Murayama, P. J. Steinhardt, Phys. Rev. Lett. **93** (2004) 201301, hep-ph/0403019.
- [8] G. Lambiase, G. Scarpetta, Phys.Rev. D **74** (2006) 087504; arXiv:astro-ph/0610367;  
G. Lambiase, S. Mohanty, L. Pizza, Gen.Rel.Grav. **45** (2013) 1771, arXiv:1212.6026 ;  
G. Lambiase, S. Mohanty, A.R. Prasanna, Int. J. Mod. Phys. D **22** (2013) 1330030, arXiv:1310.8459;  
M. Fukushima, Sh. Mizuno, K.-I. Maeda, Phys. Rev. D **93**, 103513 (2016), arXiv:1603.02403;  
A. Maleknejad, arXiv:1604.06520;  
J. I. McDonald, G. M. Shore, arXiv:1604.08213;  
S.D. Odintsov, V.K. Oikonomou, Phys.Lett. B **760** (2016) 259-262, arXiv:1607.00545;  
V.K. Oikonomou, Supriya Pan, Rafael C. Nunes, arXiv:1610.01453;  
S.D. Odintsov, V.K. Oikonomou, arXiv:1610.02533.
- [9] A.D. Dolgov, M. Kawasaki, Phys. Lett. **573**, 1 (2003);  
A.V. Frolov, Phys. Rev. Lett. **101** (2008) 061103, arXiv:0803.2500;  
E.V. Arbuzova and A. D. Dolgov, Phys. Lett. B **700** (2011) 289, arXiv:1012.1963;  
L. Reverberi, Phys. Rev. D **87**, 084005 (2013), arXiv:1212.2870.